Suspended Bicore.

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Abstract:

First the suspended bicore is described. Although the circuit-layout of this symmetric oscillator is quite simple – three passive and two active components – its behaviour is rather complex. The equations for a suspended bicore with ideal inverters and equal capacitors will be derived and with them the influence of noise on the circuit will be explained. Then the suspended bicore with different capacitors will be discussed, and the limit of this case – only one capacitor – will be explained. Finally, with knowledge of the suspended bicore, a master-slave dual bicore, which is a coupled oscillator, will be explained.

The suspended bicore:

The circuit of an Nv-neuron, as introduced by M.W.Tilden, is drawn in fig. 1. This is a pulse-delay-circuit the behaviour of which is described in 'Controller for a four legged walking machine' [1]. This Nv-neuron can serve as one part of a chain in which a pulse can circulate [2], generating a oscillatory behaviour of the output-voltage of the inverters.

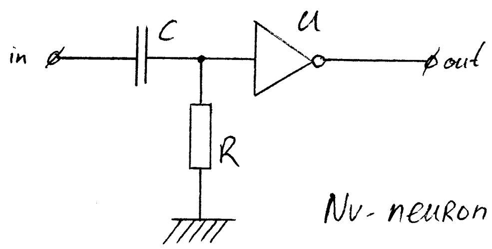


Figure 1: The Nv-Neuron.

When two Nv-neurons are connected as shown in fig. 2a the circuit is called a bicore. The neurons form a ring-like structure which generates an oscillatory output-voltage with a period determined by the capacitors and the resistors.

$$\tau_{total} = \tau_1 + \tau_2 \quad (1)$$

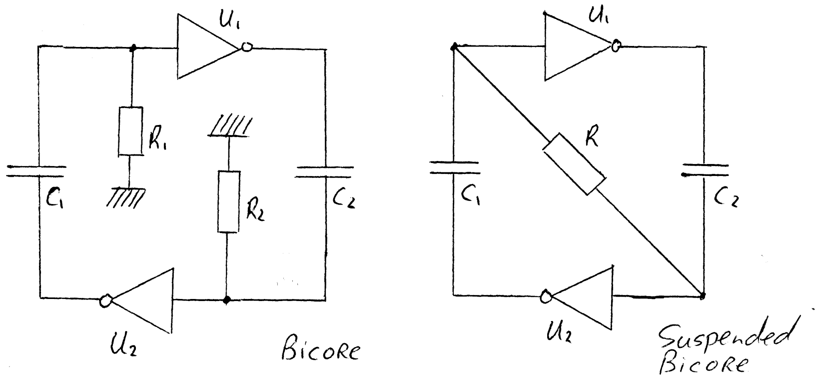
In which

$$\tau_i = \alpha \cdot R_i C_i \quad (2)$$

with $i = 1,2$

τ has the dimension of time and is determined by the resistance R in Ohms and the capacitance C in Farads; α is a constant, determined by the characteristics of the inverter. From now the capacitance will be assumed constant and equal for all capacitors to be used, unless mentioned otherwise. This gives no restrictions since the resistance alone is needed to vary the time constants of the circuits described.

If $R_1 = R_2$ the oscillation will have a duty-cycle of 50% and thus be symmetric.



(a) (b)

Figure 2: (a) the normal bicore, a ring like structure of two Nv-neurons, and (b) the suspended bicore.

In figure 2b the schematic of a suspended bicore is shown. As one can see the resistors in figure 2a are disconnected from ground, then connected to each other and replaced by a single resistor. Doing this results in a circuit which has a high symmetry.

The suspended bicore works quite differently from the normal bicore. To gain an understanding of the suspended bicore, and later the master-slave dual bicore, it could be useful to write down a set of equations for the circuit. Prior to this the oscillation is described in terms of the different parts of one period of oscillation. For that the names for certain voltages need to be defined first (fig. 3): V_{11} and V_{10} are the input- and output-voltage of inverter U1 respectively and V_{21} and V_{20} are the input- and the output-voltage of inverter U2 respectively.

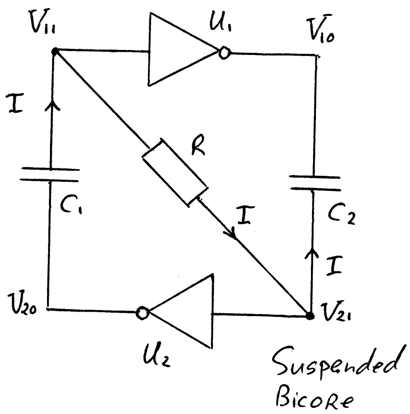


Figure 3: Direction of current *I* and names for voltages, which are used in the text.

Let V_{11} = 0 V and $V_{21} = V_{cc}$ be the supply-voltage. Because of the action of the inverters V_{10} will be equal to V_{cc} and V_{20} will be equal to 0 V. There are no voltage-differences across the capacitors, but the voltage-difference across the resistor R is V_{cc}, so a current will be flowing, charging the capacitors (see fig. 4). At a certain point the voltage across the resistor is almost zero and V_{11} and V_{21} will near the threshold-voltage of the inverters, $V_{cc}/2$.
One of the inverters will start to change state first [3], for example U1: V_{10} will start to go from V_{cc} to 0 V. Consequently, V_{21} will decrease as well, since it is coupled to V_{10} via C_2, and U2 will also change state. The output of U2 is coupled via C_1 to the input of U1, so this in turn will accelerate the change of state of U1. The result is that $V_{10} = (-V_{cc}/2)$ and $V_{20} = (3V_{cc}/2)$, but if it is assumed that the inverter only allows the input-voltages between the boundaries set by the supply-voltage levels then V_{10} = 0 V and $V_{20} = V_{cc}$ (The assumption is quite reasonable since most inverters have a protection against too

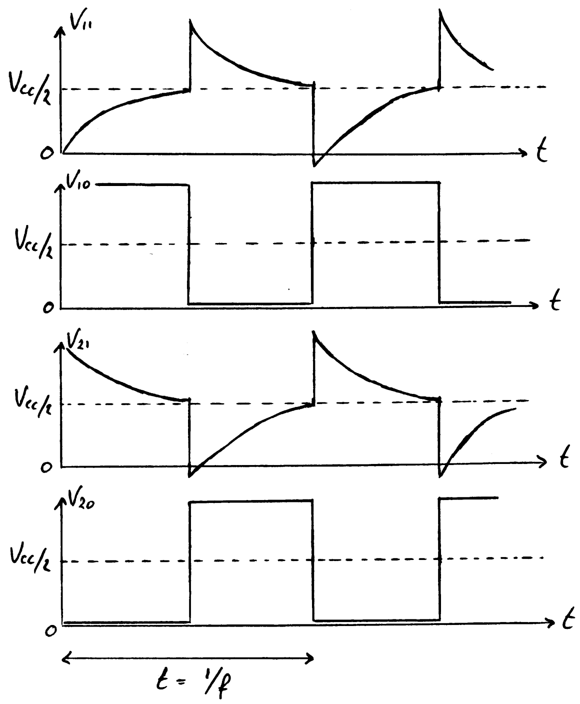


Figure 4: The waveforms V_{11}, V_{10}, V_{21} and V_{20} of the suspended bicore of fig. 3 with $C_1 = C_2$ and inverters having a threshold-voltage equal $V_{cc}/2$ for both positive and negative edges. Some noise is assumed to be superimposed on the curves so that threshold-voltages are reached. The inverters are assumed to have a threshold-voltage equal $V_{cc}/2$ for both positive and negative edges.

high or too low voltages (see fig. 5 for an example)). The described cycle will start again, with the input- and the output-voltages of U1 and U2 reversed, and at the end of it one period is completed.

Now the equations: it is assumed that no current is flowing into the inputs of the inverters (infinite input-impedance), and that output-voltage is not dependent on the current drawn (zero output-impedance). Then it is possible to define one current I flowing like shown in fig. 3.
For a capacitor the relation between the current I_c and the voltage V_c across the capacitor is a time-dependent function, given by:

$$I_c = C\frac{dV_c}{dt} \qquad (3)$$

With I_c in Amperes, C in Farads and V_c in Volts

For the current I the following relations hold:

$$I = C_1\frac{dV_1}{dt} \text{ with } V_1 = V_{20} - V_{11} \qquad (4a)$$

$$I = -C_2\frac{dV_2}{dt} \text{ with } V_2 = V_{10} - V_{21} \qquad (4b)$$

And Ohm's Law for the resistor:

$$I = \frac{V_{11} - V_{21}}{R} \qquad (4c)$$

For a time-interval, in which the inverters do not change state, V_1 can be replaced by $(-V_{11})$ and V_2 by $(-V_{21})$, since the time-derivatives of V_{20} and V_{10} are zero in this particular interval.

Making use of the assumption that $C_1 = C_2 = C$, substituting (4c) in (4a) and (4b) and rewriting gives:

$$-\frac{dV_{11}}{dt} = \frac{V_{11} - V_{21}}{RC} \qquad (5a)$$

$$\frac{dV_{21}}{dt} = \frac{V_{11} - V_{21}}{RC} \qquad (5b)$$

This is a coupled system of two first order differential equations. If it is transformed from the initial basis $\{V_{11};V_{21}\}$ to a new basis $\{V_{11}+V_{21};\ V_{11}-V_{21}\}$, the system becomes decoupled:

$$\frac{d(V_{11} - V_{21})}{dt} = -2\frac{(V_{11} - V_{21})}{RC} \qquad (6a)$$

$$\frac{d(V_{11} + V_{21})}{dt} = 0 \qquad (6a)$$

These equations are easily solved:

$$V_{11} - V_{21} = 2B \cdot \exp\left(-2(t - t_0)/RC\right) \qquad (7a)$$

$$V_{11} + V_{21} = 2A \qquad (7b)$$

In which A and B are constants and the factors 2 are there for convenience, t_0 is the time at which the exponential curve starts. From (7a) and (7b) the following equations result for a time-interval in which the inverters do not change state:

$$V_{11} = A + B \cdot \exp\left(-2(t - t_0)/RC\right) \qquad (8a)$$

$$V_{21} = A - B \cdot \exp\left(-2(t - t_0)/RC\right) \qquad (8b)$$

Note that the sign of B changes every half period, because of new the boundary-conditions which determine A and B as can be seen in fig. 4.

For example, for an ideal inverter, which has a threshold-voltage of $V_{cc}/2$ for both positive and negative edges and does not allow the input to exceed boundaries set by the supply-voltages levels 0 V and V_{cc}, $A = B = V_{cc}/2$.
The suspended bicore with this ideal inverter has one very remarkable feature: it does not oscillate; the exponential curves V_{11} and V_{21} never reach the threshold-voltage $V_{cc}/2$. This shows how very important noise and non-ideal properties are. If the exponential curve has relatively small noise superimposed on it, the inverter-input will pass the threshold-voltage and thus the circuit will oscillate. The period of oscillation however, is not constant since the circuit oscillates because of the stochastic noise. The noise and the non ideal properties of the components are not negligible in most cases.

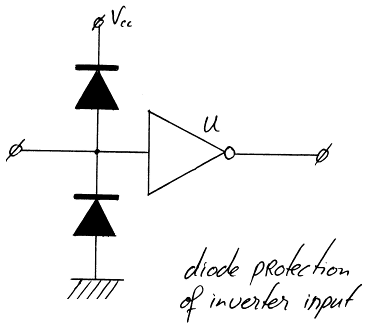


Figure 5: Schematic diagram to illustrate how the 74HC240 has internal diode protection against too high or too low voltages at its input.

For example: a suspended bicore made with inverters of the 74HC240. These inverters do allow the inputs to exceed supply-voltage levels, but only to a small extend: the inputs are protected with two diodes as shown in fig. 5. They allow the input voltages in the range ($-V_{diode}$) ... ($V_{cc} + V_{diode}$), in which V_{diode} is the threshold-voltage of the diode, which is approximately 0.6 V. Also, the threshold-voltage of the inverter at which the output starts to change is about $V_{cc}/2$ + 25 mV for the negative edges and $V_{cc}/2$ - 25 mV for the positive edges if V_{cc} = 5 V (The voltage-gain of the 74HC240 is approximately 100). For the 74HC240 the constants *A* and *B* in (8a) and (8b) are: $A = V_{cc}/2$ and $B = V_{cc}/2 + V_{diode}$. Thus the noise which has an amplitude in the mV-range has an influence on the oscillation-period. Without the noise this period is calculated to be $4.8 \cdot RC$, for V_{cc} = 5 V; with the noise this is shorter.

Normally C_1 will not be equal to C_2. This could be due to the fact that no two capacitors are exactly equal or because C_1 and C_2 are to be different. If $C_1 \neq C_2$ then the voltages on either sides of the resistor will not converge to $V_{cc}/2$, but one side will converge to a higher voltage and the other side to a lower voltages (see the dashed curves in fig. 6a). This implies that one of the inverters will reach its threshold-voltage sooner than it would in the case of equal capacitors. If one inverter reaches its threshold-voltage the output will start to change and the change will influence the input of the other inverter via the capacitor in between. So that inverter will change state as well. Noise will have less influence on the circuit if the capacitors are not equal: the gradient of the input-voltage of the inverter which initiates the change of state is bigger near the threshold-voltage than it is in the equal capacitor case.
It is important to realise that if $C_1 \neq C_2$ the duty-cycle of the oscillation will still be 50%.

This more general case, in which $C_1 \neq C_2$ can also be described by equations. The derivation is analogue to the previous derivation with $C_1 = C_2$; the differential equations describing the circuit are:

$$-\frac{dV_{11}}{dt} = \frac{V_{11} - V_{21}}{RC_1} \tag{9a}$$

$$\frac{dV_{21}}{dt} = \frac{V_{11} - V_{21}}{RC_2} \tag{9b}$$

Again this is a coupled system of two differential equations, which becomes decoupled by transforming it to another basis:

$$\left\{ \frac{2C_1}{C_1 + C_2} V_{11} + \frac{2C_1}{C_1 + C_2} V_{11} \,;\, V_{11} - V_{21} \right\}$$

Note that the basis chosen in the previous derivation with $C_1 = C_2 = C$ is a special case of this basis.

$$\frac{d(V_{11} - V_{21})}{dt} = -\frac{C_1 + C_2}{RC_2C_2}(V_{11} - V_{21}) \tag{10a}$$

$$\frac{d}{dt}\left(\frac{2C_1}{C_1 + C_2}V_{11} + \frac{2C_2}{C_1 + C_2}V_{21}\right) = 0 \tag{10a}$$

The solutions of these equations are:

$$V_{11} - V_{21} = \frac{C_1 + C_2}{C_1C_2}B \cdot \exp\left(-\frac{(C_1 + C_2)}{RC_1C_2}(t - t_0)\right) \tag{11a}$$

$$\frac{2C_1}{C_1 + C_2}V_{11} + \frac{2C_2}{C_1 + C_2}V_{21} = 2A \tag{11b}$$

In which A and B are constants and the factors $(C_1 + C_2)/C_1C_2$ and 2 are there for convenience; t_0 is the time at which the exponential curve starts. From (11a) and (11b) the following equations result for a time-interval in which the inverters do not change state:

$$V_{11} = A + \frac{B}{C_1} \cdot \exp\left(-\frac{(C_1 + C_2)}{RC_1C_2}(t - t_0)\right) \tag{12a}$$

$$V_{21} = A - \frac{B}{C_2} \cdot \exp\left(-\frac{(C_1 + C_2)}{RC_1C_2}(t - t_0)\right) \tag{12b}$$

It should be noted that in general both A and B are different for the two parts in which one period of oscillation can be divided.

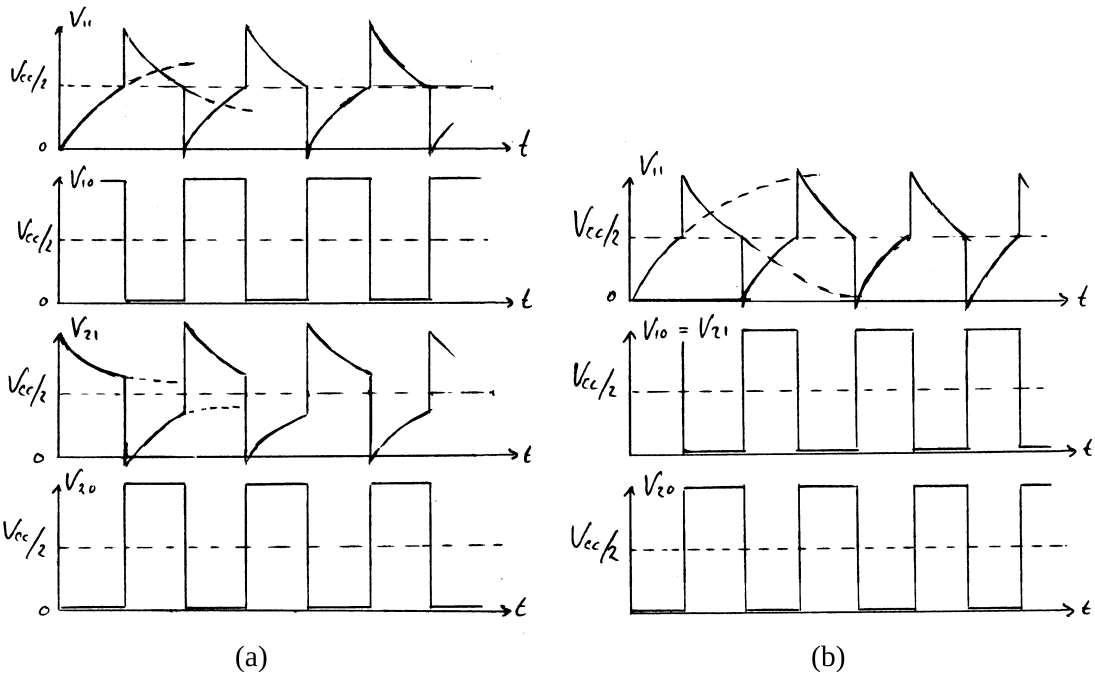


Figure 6: (a) Waveforms of the suspended bicore with unequal capacitors; $C_1 < C_2$. (b) Waveforms of the suspended bicore with only C_1; instead of C_2 there is a short-circuit – this is the limit of the case with unequal capacitors of (a). The inverters are assumed to have a threshold-voltage equal $V_{cc}/2$ for both positive and negative edges.

To illustrate how the values of A and B can be determined it is useful to examine the suspended bicore with ideal inverters: the inverters have a threshold-voltage $V_{cc}/2$ and do not allow the inputs to exceed the boundaries set by the supply-voltage levels 0 V and V_{cc}.
Let V_{11} be equal to 0 V and V_{21} be equal to V_{cc} (see fig. 6a). From equation (12a) and (12b) immediately follows for $(t - t_0) = 0$:

$$A + \frac{B}{C_1} = 0 \text{ and } A - \frac{B}{C_2} = V_{cc} \quad (13)$$

It should be noted that either A or B needs to be negative. Furthermore V_{11} and V_{21} will converge to A further in time and both V_{11} and V_{21} need to stay in the interval set by the inverter-inputs: 0 V and V_{cc}. It is evident that $A > 0$ and that $B < 0$. From (13) then results:

$$A = \frac{V_{cc}C_2}{C_1 + C_2} \text{ and } B = -\frac{V_{cc}C_1C_2}{C_1 + C_2} \quad (14)$$

In the second half of one period of oscillation C_1 and C_2 are replaced by each other and B will have the opposite sign.

If one of the capacitors is left out, (replaced by a short circuit), the resulting circuit can be seen as a limit of the suspended bicore with unequal capacitors, for which the equations (12a) and (12b) have been derived.
The oscillation of this circuit with one capacitor is shown in fig. 6b and can be understood as follows: recalling equation (3) it is clear that a short circuit can be interpreted as an infinite capacitance – no matter what value I has, there is no change in the voltage across the short circuit. Let C_2 in fig. 3 be replaced by a short circuit, then in the limit of C_2 to infinity equation (12b) reduces to:

$$V_{21} = A \quad (15)$$

To see what happens to equation (12a) it is important to realise that:

$$\lim_{C_2 \to \infty} \frac{C_1 + C_2}{RC_1C_2} = \frac{1}{RC_1}$$

so equation (12a) reduces to:

$$V_{11} = A + B \cdot \exp\left(-\frac{(t - t_0)}{RC_1}\right) \quad (16)$$

This is also the equation describing a single Nv-neuron in a ring-like structure.

The Master-Slave Dual Bicore

When the normally grounded sides of the resistors of a normal bicore are connected to the outputs of a suspended bicore, a circuit is formed which is called master-slave dual bicore (fig. 7). The master is the suspended bicore, which oscillates unaffected by the slave. The slave however is affected in its oscillation by the master via the coupling resistors R_3 and R_4.

To explain the action of the master-slave dual bicore, names for certain voltages are defined as shown in fig. 7: V_{10}, V_{11}, V_{20} and V_{21} as already defined for the suspended bicore in fig. 3 and V_{30}, V_{31}, V_{40} and V_{41} for the voltages of the outputs and the inputs of the inverter U3 and U4.

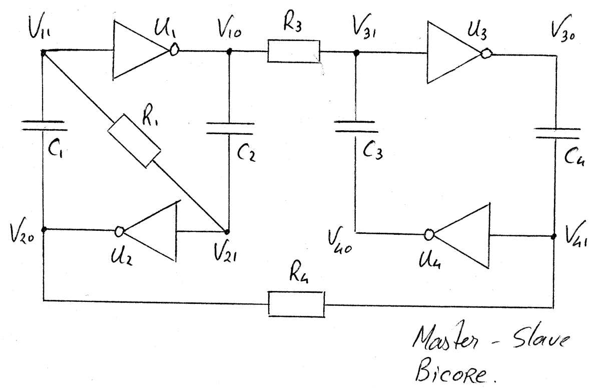


Figure 7: The Master-Slave Dual Bicore, with the names for the voltages which are used in the text.

Let V_{10} = V_{cc}, V_{20} = 0 V, V_{31} = 0 V and V_{41} = V_{cc} as at t = 0 in fig. 8. Then the voltages across the capacitors C_3 and C_4 are zero, but the voltages across the coupling-resistors R_3 and R_4 are not, so currents will be flowing through these resistors. For example V_{31}: this voltage will exponentially rise from 0 V to V_{cc} as in the second graph of fig. 8, according to the relation:

$$V_{31} = A - B \cdot \exp\left(-t \middle/ R_3 C_3 \right) \tag{17}$$

However, at Δt after the start of the exponential curve it will reach the threshold-voltage of inverter U3 and the inverter will start to change state: V_{30} will start to go from V_{cc} to 0 V. Doing this it takes V_{41} down with it, because of the coupling via C_4. In turn V_{40} will go from 0 V to V_{cc}, speeding up the changing of state of U3 via C_3. When this has happened the slave is at rest: no voltages across the capacitors and no voltages across the coupling-resistors. This state of rest continues until the master inverters reverse states, which will create voltage-differences across the coupling-resistors. A current will start to flow and the process starts all over, with reversed voltage levels.

When the circuit oscillates in this manner the frequency of oscillation of the slave is equal to and determined by the frequency of oscillation of the master: the slave waits for the master during the horizontal parts of the graphs of V_{31} and V_{41} of fig. 8. The phase difference $\Delta\varphi$ between V_{20} and V_{30} (between the master and the slave) is determined by the time Δt and by the frequency of oscillation:

$$\Delta\varphi = \frac{\Delta t}{1/f} \cdot 2\pi = \Delta t \cdot f \cdot 2\pi \tag{18}$$

In which Δt can be calculated from (17): for the circuit with inverters having a threshold-voltage equal $V_{cc}/2$ and an input-voltage bounded in the interval between 0 V and V_{cc}, the time-delay Δt is:

$$\Delta t = \ln(1/2) \cdot RC \tag{19}$$

In which $R = R_3 = R_4$, the resistance of the coupling-resistors in Ohms and $C = C_3 = C_4$ the capacitance of the slave capacitors in Farads.

It should be noted that for the phase difference between V_{20} and V_{40}, π needs to be added to $\Delta\varphi$ (V_{40} is in anti-phase with V_{30}).

By varying the frequency of the master (for example by changing R in fig. 7) both the frequency of the slave and the phase difference $\Delta\varphi$ between the master and the slave are varied.

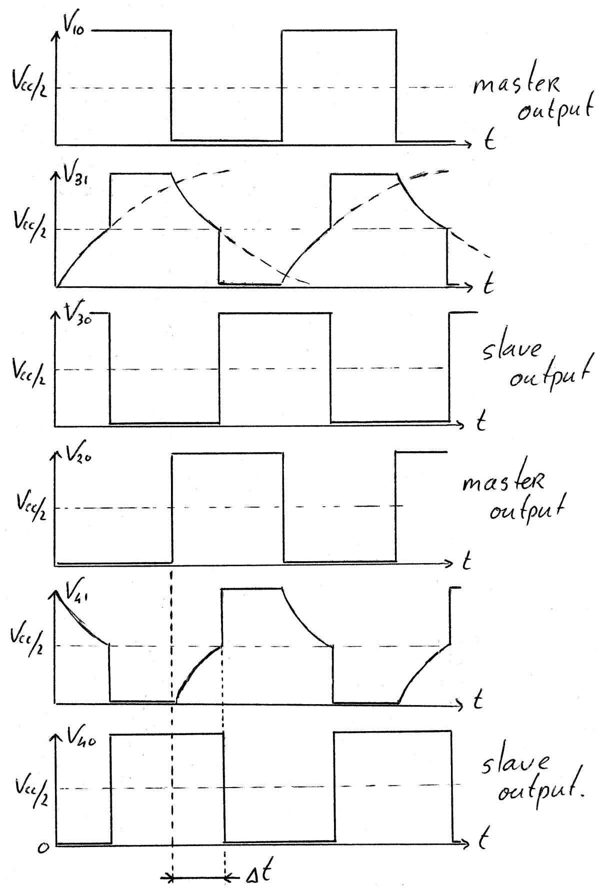


Figure 8: The waveforms V_{10}, V_{31}, V_{30}, V_{20}, V_{41} and V_{40} of the master-slave dual bicore of fig. 7. In the second graph $V_{31}(t)$ the exponential curves are dashed to give an idea of how the whole curve looks like. The time delay between the master and the slave is Δt. The inverters are assumed to have a threshold-voltage equal $V_{cc}/2$ for both positive and negative edges.

The frequency of oscillation of the slave can differ from the frequency of oscillation of the master: this happens if half of the period of the master is shorter than the Δt caused by the exponential curves of the slave-voltages (if $\Delta\varphi$ in equation (18) is bigger than π). Contrary to the previous case, in which the slave had to wait for the master, the slave is now too slow for the master. An oscillation like this is shown in fig. 9: the slave starts with an exponential rise of V_{31}, due to a voltage across the coupling-resistor. However, at a certain moment the output of the master will reverse so the voltage across the coupling-resistor will change sign and V_{31} will decrease. Again the master-output reverses so V_{31} rises; this will happen until V_{31} reaches the threshold-voltage of the slave-inverter. The slave-inverter changes state, and via the capacitor the change causes the other slave-inverter to change state as well. The cycle will start again, only with the voltage-levels reversed.

Conclusion

The suspended bicore of fig. 3 shows a very interesting behaviour in response to noise: if the inverters are ideal and the capacitors equal, the oscillation-period of the circuit is very sensitive to noise. This sensitivity is decreased by using inverters which have a threshold-voltage, which is not exactly in the middle of maximum and minimum input-voltages, or by using capacitors which are not equal. These two ways of decreasing the sensitivity of the circuit are based on the fact that the exponential curves of the input-voltages of the inverters have a larger gradient near the threshold-voltage of the inverters. Consequently the relatively small noise that is superimposed on these exponential curves will have less influence on the exact inversion-moment, and thus less influence on the period of oscillation.

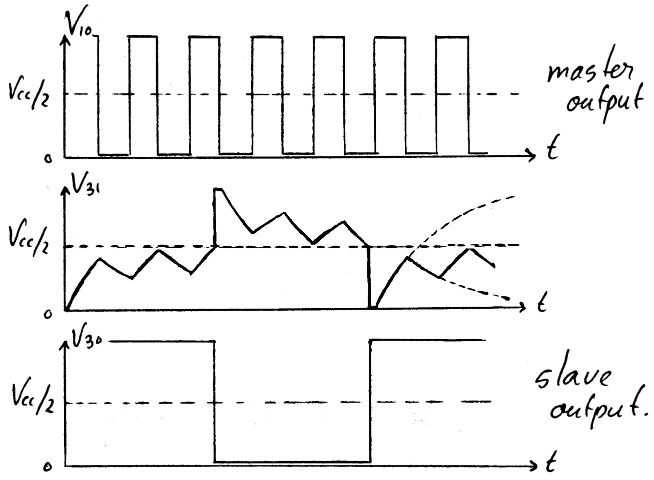


Figure 9: The waveforms V_{10}, V_{31}, and V_{30}, of the master-slave dual bicore of fig. 7, when the period of oscillation of the master is to fast for the slave to follow. The waveforms V_{20}, V_{41} and V_{40} are not shown: they are the same, but inverted around $V_{cc}/2$ if both coupling-resistors are equal.

If the circuit is expanded to a master-slave dual bicore (fig. 7) a coupled oscillator results. For such an oscillator the frequency of oscillation of the master is equal to that of the slave if the natural oscillation-period of the slave is smaller than the oscillation-frequency of the master. The phase-difference between the master and the slave can be controlled by varying the oscillation-frequency of the master alone. This however can only be done to a certain extend since for a frequency which is too high the slave is not able to follow the master anymore: its frequency will then be smaller than the frequency of the master.

A possible application for the master-slave dual bicore is as driving-circuit for a two-motor walking robot such as the one described in 'Controller for a four legged walking machine' [1] and 'Coupled Oscillators and Walking Control' [4]. The circuit could also be used as a central pattern generator (CPG) in more advanced mobile robots which are inspired by biological systems.
The controller for the robot could be designed so that for example motor-noise is coupled back into the master-slave dual bicore via an appropriate filter. This way both the frequency of oscillation and the phase-difference between the master and the slave will adapt to the properties which influence the level of noise, like the load on the motor. The oscillator-circuit itself can be designed to have a certain sensitivity to this noise.

It should be noted that in this text the output-impedance of the inverters is assumed to be zero. Generally this however is not the case and will have an effect on the behaviour of the circuit if a load is applied.

Literature

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